Learning Objectives

The student should develop an understanding of what trigonometric ratios essentially are through this lesson. It builds step-by-step from prior knowledge about similar triangles, through discovering equivalent ratios, to the formal definitions of sine, cosine, and tangent.

Math Objectives

- The student will learn the basic right-triangle definitions for sine, cosine, and tangent.
- The student will understand the connections between trigonometric ratios and their prior knowledge about similar triangles.

Technology Objectives

- The student will construct right triangles with given acute angle measures in Geometry Expressions.

Math Prerequisites

- The student should understand what makes similar triangles, and the proportionality of their sides.

Technology Prerequisites

- None. A basic understanding of Geometry Expressions helps, but is not required.

Materials

- A computer with Geometry Expressions for each student or pair of students.
- A scientific calculator.

Overview for the Teacher

This is an introduction, not a full lesson. It allows students to build an understanding of sine, cosine, and tangent as simply names for specific ratios of sides of similar right triangles. While this is a fairly common exercise to do with straightedge and protractor, the precision of student constructions is often not adequate for them to see the patterns. There could be a great deal of variation in the computed value for sine of a specific angle measure, for example. Using Geometry Expressions allows for the necessary precision.

1) All the triangles should be the same shape, but different sizes – i.e. they should be similar.
2) Student answers will vary, but rows 4, 5, & 6 should each have the same answer in all four columns. These should correspond to the sine, cosine, and tangent values of the student's chosen angle measure.
3) Students should get the same answer all the way across each row.
4) Angle-Angle Similarity theorem. This is a good small group discussion question. Have students review the theorem, and possibly the other similarity theorems while they're at it. If they haven't yet learned all the similarity theorems, this provides a sneak-preview of the topic, and you can discuss similarity in more general terms.
5) Answers will vary.

Students will need some guided and independent practice with the vocabulary at this point. Since such exercises are in virtually every Algebra 2 textbook, they are not reproduced here. Some suggested exercises would include:

- Given a right triangle and a specified angle, students name sides as opposite, adjacent, and hypotenuse. Use a variety of orientations.
- Given right triangles and all three sides, students produce sine, cosine, and tangent ratios for both acute angles.
- Students use a calculator to determine sine, cosine, and tangent ratios for given angles.
- Given a right triangle, all three sides, and a ratio, students determine if it is sine, cosine, or tangent for a given angle.
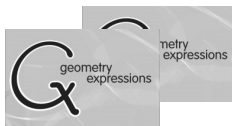
Date: _____

Right Triangle Trigonometry Intro

Pick a number between 10 and 80. Write it down here. _____

In Geometry Expressions, select File/New
Edit/Preferences/Math/Math/Angle Mode – set to degrees.
You are going to create a right triangle with one angle measure equal to the number you chose.

- Use **draw line segment** or **draw polygon** to create triangle ABC.
- Select angle C (Shift and click on AC, then BC), and use **constrain perpendicular** .

- Select angle A, and use **constrain angle** ; set your angle measure equal to the number you chose at the beginning.

- Highlight AB, then **calculate real distance** . This should give you a $Z_0 \Rightarrow$ and a number. Repeat with line segment BC and AC, which will give Z_1 and Z_2. These are the lengths of the line segments.

1) Now drag on any vertex of the triangle. What you notice about the size and shape of the triangle?

2) Complete the following table for four different versions of the triangle you constructed. They should all have the angle measure you chose at the beginning. When you get to the ratios, round to the thousandths place.

		Triangle #1	Triangle #2	Triangle #3	Triangle #4
1	Length AB				
2	Length AC				
3	Length BC				
4	$\dfrac{BC}{AB}$				
5	$\dfrac{AC}{AB}$				
6	$\dfrac{AC}{AB}$				

3)   What do you notice about the rows 4-6?

4)   Why is it safe to say that all triangles ABC with a right angle at C and your chosen angle measure at A will be similar to each other (i.e. each is a scale model of all the others)?

Since all right triangles with a given angle measure will be similar to each other, the ratios of their sides will be equal. This is what you saw in the last three rows. All of trigonometry is based on these facts about similar triangles and equal ratios. Because there are many, many applications and uses of the ratios of the sides of right triangles, mathematicians have given them specific names.

- Row 4 is called **Sine**: the ratio of the side opposite the angle to the hypotenuse.
- Row 5 is called **Cosine**: the ratio of the side adjacent to the angle to the hypotenuse.
- Row 6 is called **Tangent**: the ratio of the side opposite the angle to the adjacent side.

5)   Use your calculator to determine the sine, cosine, and tangent of the angle measure you chose.   Compare to the values in your table.
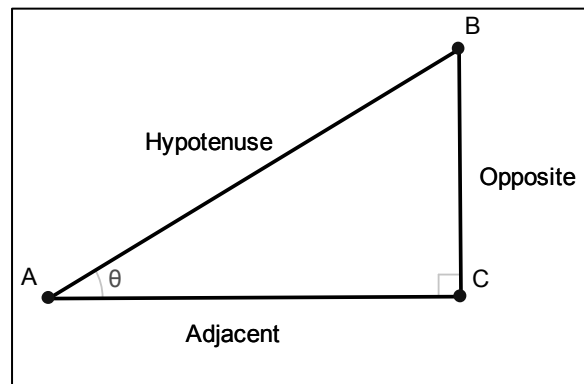Sine:

Cosine:

Tangent:

A common memory trick to use is SOH-CAH-TOA.

$$Sine = \frac{Opposite}{Hypotenuse} \qquad SOH$$

$$Cosine = \frac{Adjacent}{Hypotenuse} \qquad CAH$$

$$Tangent = \frac{Opposite}{Adjacent} \qquad TOA$$

Learning Objectives

This lesson is designed to transition students from an understanding of right triangle trigonometry to an understanding of the unit circle and trig ratios for angles greater than $90°$ or less than $0°$. They should develop a solid conceptual connection between right triangles and the unit circle, as well as understand the expanded definitions for sine, cosine, and tangent.

Math Objectives

- The student will understand the connections between the right triangle trigonometry and coordinates of a point on a unit circle.
- The student will understand and be able to use the expanded definitions of sine, cosine, and tangent for angles greater than $90°$ or less than $0°$.

Technology Objectives

- The student will become familiar with use of the various tools in the Geometry Expressions program, including drawing, constraining, animating, and constructing a locus

Math Prerequisites

- The student must understand and be proficient in right-triangle trigonometry.
- The student must understand a four-quadrant Cartesian coordinate system.

Technology Prerequisites

- Students should be able to complete this lesson with little to no background with Geometry Expressions.

Materials

- Computer with Geometry Expressions for each student or pair of students.

Overview for the Teacher

In this lesson, students will transition from an understanding of trigonometry which is limited to right triangles, to an understanding of the unit circle. This is done in a way that strongly emphasizes the logical connection and partial equivalence of the two aspects of trigonometry. Most students should be able to work through this lesson independently – it is not designed to require any direct instruction from the teacher. However, some common issues to look for and be aware of are outlined below.

This lesson involves a minimal amount of writing; most of the work will be done on the computer. You may want to institute a set of checkpoints, where you confirm the work has been done on the screen, and initial student papers. We suggest steps 8, 14, and 16 would be appropriate. Furthermore, the independent practice is not included, since exercises of this type are readily available. Following this lesson, students will need to do some practice (in worksheet form or otherwise) with…

- identifying coordinates of points on a unit circle given an angle of rotation
- identifying the sign of sine and cosine in the four quadrants
- correlating coordinates of points on a unit circle to their respective trig values.

Steps 1-5: In early versions of GX, some constraint contradictions tended to happen if one constructed the triangle by constraining point C to the axis directly or through its coordinates. If the location of C is established using a slope of 0 or direction of $0°$, the problem is avoided. This shouldn't be a problem in later versions of the software.

By step 8, student diagrams should resemble the following. (Numbers will be different depending on the angle measure chosen and the length of c.)
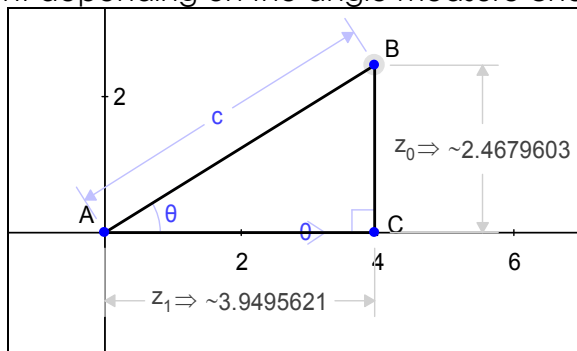
Step 14: Students may want to delete the side length measurements after they figure out the equivalence with the coordinates of point B. This is fine, and cleans up the diagram.
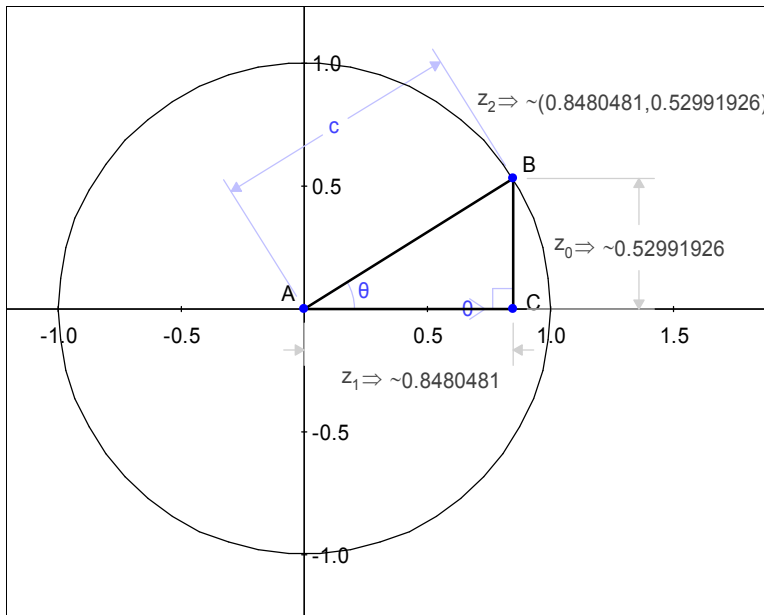
Step 16: The default variable for constructing a locus is c, but θ is needed. If students get a straight line instead of a circle, have them make sure they chose variable θ.

Final diagram should look like:

Name:

_____

Date: _____

Intro to Unit Circle Trigonometry

Before you get started:
Open a new Geometry Expressions file. Make sure you are in degree mode and display your axes. (Edit/Preferences/Math/Math/Angle Mode ->

Degrees; toggle the axis/grid button on the top toolbar ▦)

Right Triangle:
1) Pick a number between 10 and 80. Write it here: _____
2) You are going to create a triangle in the first quadrant. Use **draw line**

segment 🖉 to create triangle ABC, with A snapped to the origin, and B and C both somewhere in the first quadrant.

3) Highlight line segment AC and use **constrain slope** 🔒 to set the slope to zero. This makes the segment line up with the x-axis.
4) Highlight line segments BC and AC at the same time by holding down

the shift key and clicking on each. Use **constrain perpendicular** 🔒 to form a right angle.
5) Highlight line segments AB and AC at the same time and use **constrain**

angle 🔒 to constrain the angle measure to θ.

You have just created a right triangle in **standard position**. A right triangle in standard position has the vertex with the angle you are concerned about (θ) at the origin, one leg of the triangle on the positive x-axis, and the third vertex in the first quadrant. Note that any right triangle could be positioned this way by rotating, translating, and/or reflecting it.
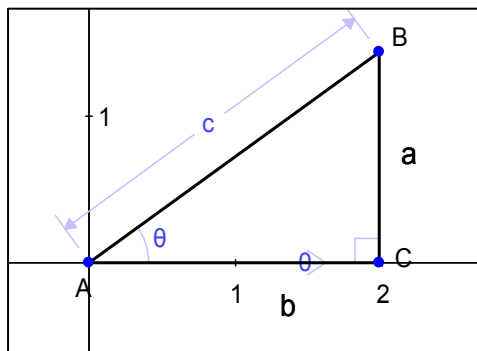
6) Highlight segment AB and use **constrain distance** 🔒 to determine its length to be the variable c.
7) Now make the triangle your own. Use the Variables tool panel, highlight θ, then below that box, type the number you
chose in part 1. Lock θ
by clicking on the 🔓 icon next to the number.

Variables		⊕ ×
Variables	Functions	
Name	Value	Locke
c	4.4958507	-
θ	32	+
	Ⅲ	▶
θ	32	🔒

8) Use **calculate real distance** 🔲 to calculate lengths *a* and *b* of your triangle; length *c* is displayed in the Variables window. Use those lengths to determine your sine, cosine, and tangent ratios.

Side	Length
a	
b	
c	

|	Ratio	Decimal
Sine (θ)	/	
Cosine (θ)	/	
Tangent (θ)	/	

9) Use the trig functions on your calculator to check the ratios. Make sure your calculator is in degree mode.

10) What should happen to the triangle if we increase or decrease length *c*? Test your idea on GX by highlighting *c* in the Variables tool panel and changing the value. You can increase or decrease *c* by either typing in new numbers, or using the scroll bar underneath.

11) What will happen to the trig ratios as you do this?

12) For the rest of the activity, set *c* equal to 1 and lock it in the Variables tool panel. You will probably need to scale up (🔍 button on the top tool bar – like zooming in) so the diagram takes up much of your screen.

13) Highlight point B and use **calculate real coordinates** to find the location of B. Compare this to your results in step 8. What do you notice?

14) Now adjust your diagram for some other angle measures by unlocking θ (click on the icon again), then either using the Variables tool panel, or clicking and dragging point B in your diagram. Test whether the trig ratios for two different angle measures correspond to coordinates for point B.

θ	Coordinates of B	Cos(θ)	Sin(θ)
	(,)		
	(,)		

Why does this work? If a right triangle is in standard position, the opposite side length is equal to the y-coordinate of point B, and the adjacent side length is equal to the x-coordinate of B.
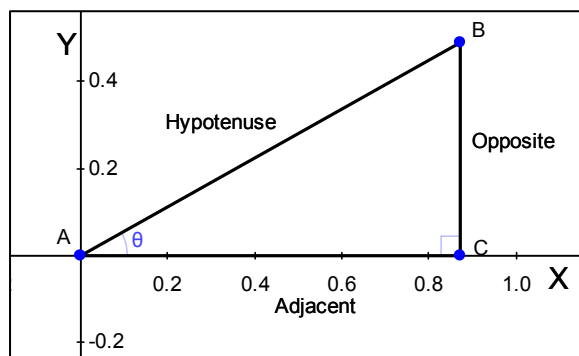
Also, notice that we scaled the triangles back so that the hypotenuse is always 1. This can be done to any right triangle without changing the ratios of the sides.

Therefore,

$$sin\theta = \frac{opposite}{hypotenuse} = \frac{y}{1} = y$$

$$cos\theta = \frac{adjacent}{hypotenuse} = \frac{x}{1} = x$$

$$tan\theta = \frac{opposite}{adjacent} = \frac{y}{x} = \frac{Sin\theta}{Cos\theta}$$

These facts can be used to expand the definitions of sine, cosine, and tangent to angle measures greater than $90°$ and less than $0°$, as we will see shortly.

15) We want to look at all the possible values of θ. To do this, highlight θ in the Variables tool panel, and then look at the animation tools at the bottom of that section.

These tools allow you to move continuously through a range of values for a given variable. The bottom left and right boxes indicate the minimum and maximum values for θ. We want to see between $0°$ and $360°$. You can manually run through those values by
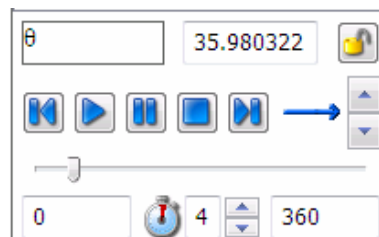
clicking and dragging the scroll bar, or have the computer do it automatically by clicking on the play button. Notice the path that point B takes.
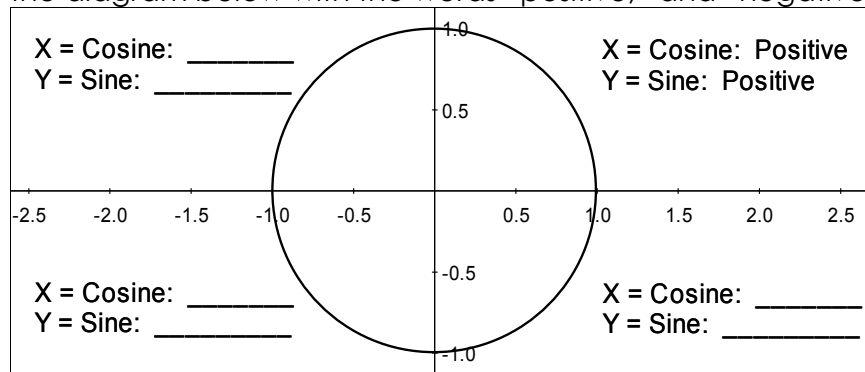
16) To see the pathway more clearly, highlight point B, and use the

construct locus ![tool icon] tool. You'll see a pop-up menu; make sure it is set to variable θ, start value 0, and end value 360. Click OK. Then repeat the animation from step 15.

When you consider the values of $0°$ through $90°$ - all the possibilities for a right triangle - the point B makes an arc. As you continue the pattern beyond the triangle, you create a circle with radius one, centered at the origin, which is called a **unit circle**. The coordinates of B, as we saw earlier, are $(\cos\theta, \sin\theta)$, where θ is the angle made with the positive x axis. The trick here is to think of θ a little differently. Instead of just being an acute angle of a right triangle, think of θ as the number of degrees that point B has rotated through since leaving the x-axis in the animation. For measures between $0°$ and $90°$, these two definitions of θ are exactly the same. However, the second definition allows us to use angle measures greater than $90°$.

17) Similarly, angle measure of less than $0°$ indicate a rotation in the opposite direction. Adjust your beginning value in the variables menu to –360, and use the scroll bar to investigate.

In short, the expanded definitions of sine and cosine are the coordinates of a point on a unit circle, after the point has rotated θ degrees counterclockwise from the positive x-axis. The coordinates are $(\cos\theta, \sin\theta)$. Similarly, tangent of θ becomes $\dfrac{y}{x}$.

18) Pause the animation at various angle measures in the different quadrants and determine from the coordinates where sine is positive and negative, and where cosine is positive and negative. Complete the diagram below with the words "positive," and "negative."

X = Cosine: _____
Y = Sine: _____

X = Cosine: Positive
Y = Sine: Positive

X = Cosine: _____
Y = Sine: _____

X = Cosine: _____
Y = Sine: _____

19)   Based on what you have learned so far, or by observing the points on
        the unit circle, what is the greatest and least value you could get for
        sine of some angle θ? This is called the range of sine.

20)   What is the greatest and least value you could get for cosine of some
        angle θ? This is the range of cosine.